Unit 5 Review

Name: Key

1. Determine whether each sequence is arithmetic, geometric, or neither

a) 3, 8, 13, 18,... arithmetic $d=5$

5 5 5

b) 3, 6, 12, 24,... geometric $r=2$

c) 1, 3, 7, 15,... neither

2 4 8

d) −17, -3, 11, 25 arithmetic $d=14$

14 14 14

2. Determine the 30th term of the arithmetic sequence: 12.4, 13.6, 14.8, ... $d=1.2$

$a_{30} = 12.4 + (30-1)(1.2)$

$a_{30} = 47.2$

3. Determine the first 8 terms of the arithmetic sequence, when the 3rd term is 41 and the 7th term is 75.

$d = \frac{75-41}{7-3} = 8.5$

1	2	3	4	5	6	7	8
24	32.5	41	49.5	58	66.5	75	83.5

4. Find a formula for a_n for the arithmetic sequence.

$a_1 = 82$ $a_4 = 64$ $d = \frac{64-82}{4-1} = -6$

$a_n = 82 + (n-1)(-6)$

$a_n = 88 - 6n$

5. Determine the 10th term of the geometric sequence: 2, 6, 18, ...

$r=3$

$a_{10} = 2(3)^9$

$a_{10} = 39,366$

6. Given the following geometric recursive sequence, find the first four terms of the sequence.

$a_1 = 120$ and $a_{k+1} = 1.1a_k$

120, 132, 145.2, 159.72

7. Determine the first six terms of the geometric sequence, when the 2nd term is 75 and the 3rd term is 225.

$r = 3$

1	2	3	4	5	6
25	75	225	675	2025	6075

8. Insert 3 numbers between 6 and 1536 so the 5 numbers form a geometric sequence.

1	2	3	4	5
6	24	96	384	1536

$a_5 = 6(r)^4$

$1536 = 6r^4$

$256 = r^4$

$r = 4$

9. For the arithmetic series 3+ 7 + 11 + 15 + ... , determine S_{33} .

$S_{33} = \frac{33}{2}(3+131)$

$S_{33} = 2211$

$a_{33} = 3 + (32)4$

$a_{33} = 131$

10. For the geometric series 128 + 64 + 32 + ... , determine S_9

$r = \frac{1}{2}$

$S_9 = 128\left(\frac{1-\left(\frac{1}{2}\right)^9}{1-\frac{1}{2}}\right)$

$S_9 = 255.5$

11. Determine the sum of each series.

a) 3+12+21+...+102

$102 = 3 + (n-1)9$

$12 = n$

$\frac{n}{2}(a_1 + a_n)$

$\frac{12}{2}(3 + 102)$

$= 630$

b) 1024 + 512 + 256 + ... + 32

$r = \frac{1}{2}$

$32 = 1024\left(\frac{1}{2}\right)^{n-1}$

$n = 6$

$S_6 = 1024\left(\frac{1-.5^6}{1-.5}\right)$

$S_6 = 2016$

12. Find the sum

$$\sum_{n=1}^{6} 3n - 2$$

~~$S_6 = \frac{6}{2}(-1 + 214)$~~

~~$S_6 = 639$~~

$S_6 = \frac{6}{2}(1 + 16)$

$S_6 = 51$

13. Use sigma notation to write the sum of the following sequence.

$1(4^1) + 2(4^2) + 3(4^3) + \ldots + 8(4^8)$

$$\sum_{n=1}^{8} n(4^n)$$

14. Find the sum:

$$\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n = \frac{2}{1-\frac{2}{3}} = 6$$

$\frac{2}{\frac{1}{3}}$ $2 \cdot \frac{3}{1}$

15. Simplify the ratio $\frac{78!}{75!}$ $= 78 \cdot 77 \cdot 76 = 456{,}456$

16. Find the eighth term in the expansion of $(3a - 2b)^{10}$

$x = 3a \quad y = -2b$

${}_{10}C_7\, x^3 y^7$

$\frac{10!}{3!\,7!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$

$120\,(3a)^3(-2b)^7$

$120\,(27a^3)(-128b^7)$

$= \boxed{-414720\,a^3b^7}$

17. Write the expansion of $(x + 2)^7$

$1x^7 + 7x^6(2)^1 + 21x^5(2)^2 + 35x^4(2)^3 + 35x^3(2)^4 + 21x^2(2)^5 + 7x(2)^6 + (2)^7$

$\boxed{x^7 + 14x^6 + 84x^5 + 280x^4 + 560x^3 + 672x^2 + 448x + 128}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

18. Write the expansion of $(2x - 3y)^5$

$a = 2x \quad b = -3y$

$a^5 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$(2x)^5 + 5(2x)^4(-3y) + 10(2x)^3(-3y)^2 + 10(2x)^2(-3y)^3 + 5(2x)(-3y)^4 + (-3y)^5$

$\boxed{32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 + 810xy^4 - 243y^5}$

19. Find the coefficient of the $a^4\, b^3$ in the expansion of $(a - 2b)^7$

$x = a$

$y = -2b$

$n = 7$

$r = 3$

${}_7C_3 = \frac{7!}{4!\,3!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$

$35\,(a)^4(-2b)^3$

$= -280\,a^4b^3$